A New Determination of the π^- Rest Mass

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The π^- rest mass has been determined from measurements of X-ray transitions in pionic atoms, using muonic transitions for the calibration, to be $m_{\pi^-} = 139.571 \pm 0.010$ MeV. A new upper limit for the μ-neutrino rest mass was deduced: $m_\nu < 0.78$ MeV at 90 per cent confidence.

1. Introduction

The measurement of the energies of pionic X-rays provides at present the most accurate methode for the determination of the π^- rest mass. Precise measurements were done by Shafer [1, 2] and by Backenstoss et alii [3, 4]. Shafer used a crystal spectrometer and Backenstoss et al. applied Ge(Li) detectors with radioactive sources for the calibration. Since it is desirable to obtain several independent values of such a fundamental quantity as the π^- rest mass, we performed a new measurement with Ge(Li) detectors. We applied a new calibration method with muonic X-ray lines as the energy standard. Muonic X-ray energies can be calculated for the transitions in question with a precision of a few eV. A mixture of Cd and Y was chosen as target material, because it provides suitable pionic transitions lying between muonic transitions. During the preparation of this publication two new values of the π^-- rest mass were published [5, 6] with errors of 2.4 keV and 2.1 keV, respectively. The difference between these two new measurements of 6.4 keV indicates that still now additional measurement are necessary. The novel method applied in the present paper provides a really independent new value which, up to now, cannot discriminate between the diverging values [5, 6]. The method will, however, allow to measure the π mass with very high precision if the experiment is repeated on a modern high-intensity accelerator.

2. Experiment

The pion and muon beam came from the muon channel of the 600 MeV CERN synchrocyclotron. The experimental set-up is shown in Figure 1. Two planar Ge(Li) detectors were used [1.0 cm^3 (Det. I) and 2.8 cm^3 (Det. II) volume]. The in-beam resolution at 200 keV was 1.0 keV and 1.6 keV, respectively. The pion and muon beam had a mean rate of 10^4 stopped particles per second. The detector pulses were registered in coincidence with the $(123\bar{4})$ stop

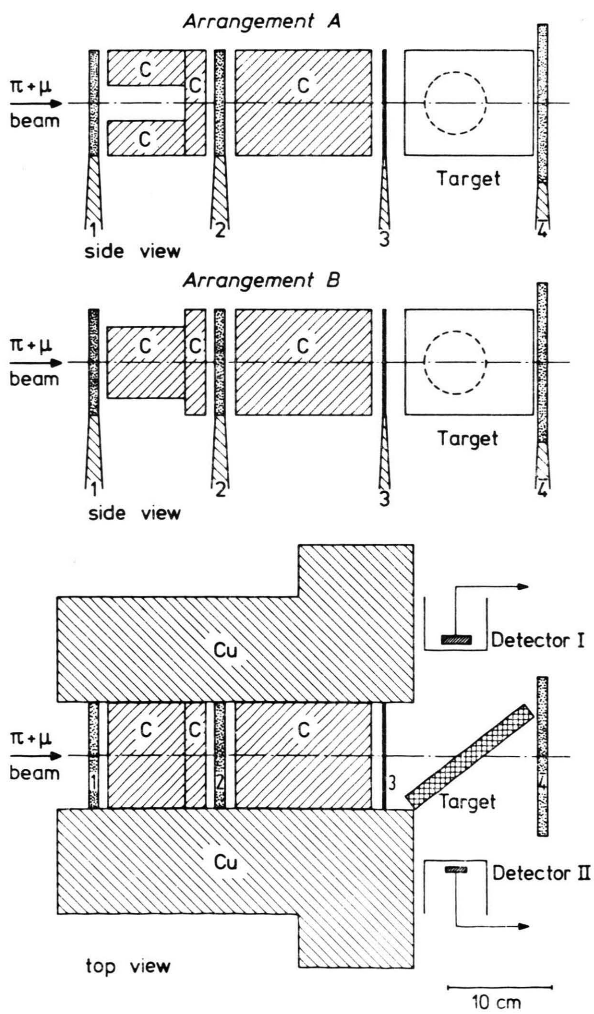

Fig. 1. Experimental arrangement. 1, 2, 3, $\bar{4}$ scintillation counters, C carbon moderator, Cu copper shielding.

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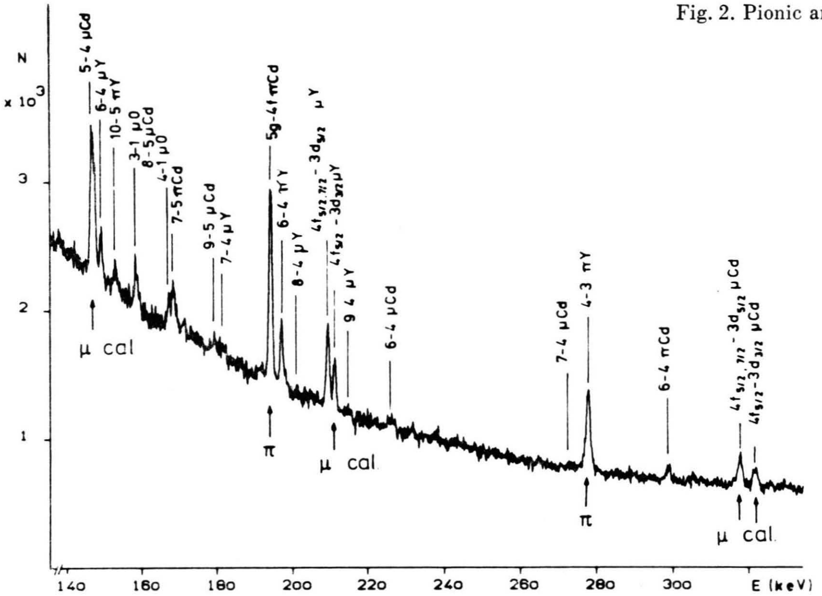

Fig. 2. Pionic and muonic spectrum of Y and Cd (spectrum IA).

signal and stored in two multichannel analyzers. The targed consisted of a homogeneous mixture of Cd and Y metal powder in a lucite container (dimensions $140 \times 100 \times 9$ mm^3). The target was inclined relative to the beam and to the detectors by 45°. The target density was 2.8 g/cm^2.

In order to obtain muonic and pionic lines with comparable intensities simultaneously in the spectrum, two special degrader arrangements were applied (Figure 1). Muons have a longer range than pions and need more degrader (carbon). Therefore, arrangement A gives more muon concentration at the upper and lower side of the target and more pion concentration at the center, whereas arrangement B enhances the muon rate at the center and the pion rate at the sides. The comparison of the spectra obtained with arrangement A and arrangement B provides an essential check, if the different distributions of pions and muons in the target influence the measured energies of the muonic and pionic X-rays. No effect was observed beyond the statistical error (cf. Section 3).

Two runs were made with collection times of about 12 hours for each run, using arrangement A and B, respectively. Therefore, four spectra were obtained with the two detectors. The spectra are labelled, for instance, I A, which means detector I and arrangement A. The spectrum I A is shown in Figure 2.

3. Evaluation and Energy Calibration

The positions of the peaks in the spectra were evaluated by means of a least squares fit computer program. The lines were assumed to have Gaussian shapes. No difference in the line shapes were observed between the pionic and muonic lines. The background was fitted by sections of straight lines. All statistically significant lines and fine structure components were taken into account. Only those lines were included in the calibration which were not disturbed by others. Table 1 gives the positions in channels of the interesting Y and Cd lines in the four measured spectra with the statistical errors (in parenthesis we give the energy values in keV of the pionic lines with the total errors).

Energy calibration was performed with theoretically calculated muonic transition energies. This method avoids same sources of systematic errors which may be present if nuclear gamma rays are used for the calibration. Nuclear gamma rays come either from an external calibration source, or from the target after muon or pion capture. In the first case they have no time correlation to the telescope signal, in the second case they may be delayed. The different coincidence conditions may cause energy shifts. Gamma rays from radioactive sources may reach the detector from different directions and after penetration of a different amount of matter, than do the X-rays from the target. The pulse height from Ge(Li) detectors may depend on the direction of the gamma rays. In our experiment pionic and muonic X-rays have almost the same geometrical origin and the influence of the difference of arrangements A and B

Table 1. Position in channels and statistical errors of the interesting lines in the Y and Cd spectrum. In parenthesis we give the final energy values in keV of the pionic lines, with total errors.

Muonic transitions	Measurement I A	Measurement II A	Measurement I B	Measurement II B
$4f_{5/2}-3d_{3/2}$ Cd	2528.59 ± 0.47	2563.14 ± 0.21	2528.63 ± 0.25	2559.80 ± 0.21
$4f_{7/2}-3d_{5/2}$ Cd	2496.65 ± 0.30	2531.73 ± 0.21	2496.68 ± 0.19	2528.94 ± 0.15
$4f_{5/2}-3d_{3/2}$ Y	1637.93 ± 0.12		1637.55 ± 0.13	1689.71 ± 0.26
$5g_{9/2}-4f_{7/2}$ Cd	1119.46 ± 0.12	1188.17 ± 0.12	1119.22 ± 0.08	1184.43 ± 0.14
Pionic transitions				
4f − 3d Y	2175.95 ± 0.13	2218.95 ± 0.12	2176.10 ± 0.51	2214.82 ± 0.34
	(278.060 ± 0.043)	(278.092 ± 0.036)	(278.109 ± 0.111)	(277.974 ± 0.077)
5g − 4f Cd	1501.86 ± 0.07	1560.90 ± 0.09	1502.03 ± 0.21	1557.75 ± 0.33
	(194.184 ± 0.022)	(194.157 ± 0.029)	(194.242 ± 0.046)	(194.214 ± 0.076)

(Fig. 1) has been checked to be less than the statistical errors. This calibration method has the additional advantage that some correction terms of the theoretical calculations have similar values for pionic and muonic atoms and, therefore, the uncertainties of these terms do not affect so much the final result. This concerns electron screening, vacuum polarization and finite size terms. The error of the muon mass can be neglected compared with the other errors.

The energies of the muonic calibration lines have been calculated with a computer program [7] including all corrections listed in Table 2. The vacuum polarization terms were taken from Fricke [8] $[\alpha(\alpha Z)]$, Blomqvist [9] $[\alpha^2(\alpha Z)$ and $\alpha(\alpha Z)^3]$ and Wichmann and Kroll [10] $[\alpha(\alpha Z)^5$ and $\alpha(\alpha Z)^7]$. The errors of these terms are generally assumed to be a few eV at most [7] and, as discussed above, tend to cancel. The Lamb shift was calculated according to Erickson [11] and Klarsfeld and Maquet [12], nuclear polarization according to Ericson and Hüfner [13] and the shift due to the anomalous magnetic moment of the muon according to McKee [14]. For the treatment of the electron screening see Section 4.

A linear energy calibration was applied with four muonic lines. Figure 3 shows the differences between the calculated energies and the values determined from the line position and adopted linear calibration. The errors bars represent the statistical errors only. This figure shows that no deviation from the linear calibration was observed within the statistical errors. The linearity of the detector was also tested with radioactive sources. A good check of the energy calibration is given by the energy of the 2p-1s muonic X-ray line of 0 which was observed in all four spectra. This line comes from the container and the calibration has to be extrapolated. The 2 − 1 μO

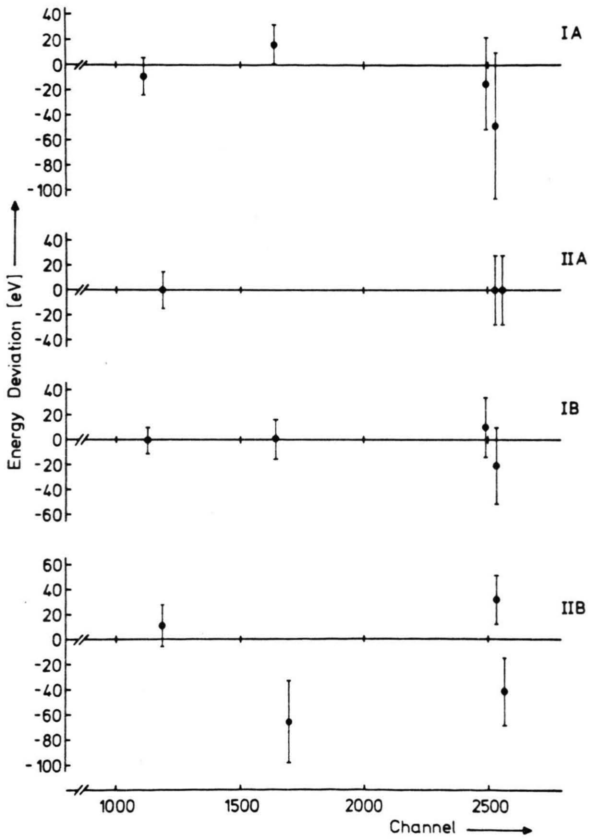

Fig. 3. Energy deviations (calculated minus fitted energy values) of the calibrations lines versus channel number.

energy was determined to be 133.512 ± 0.020 keV, in agreement with the value of 133.525 ± 0.015 keV as measured by Dubler et alias [15].

4. Determination of the π^- Rest Mass

The π^- rest mass was determined by comparison of the measured and the calculated pionic transition

Table 2. Calculated muonic and pionic transition energies and measured pionic transition energies (energies in keV).

	Muonic transitions				Pionic transitions [c]	
	$4f_{5/2}-3d_{3/2}$ Cd	$4f_{7/2}-3d_{5/2}$ Cd	$4f_{5/2}-3d_{3/2}$ Y	$5g_{9/2}-4f_{7/2}$ Cd	$4f-3d$ Y	$5g-4f$ Cd
Calculated energies	321.994 ± 0.005	317.987 ± 0.003	211.095 ± 0.002	146.615 ± 0.001	278.061 ± 0.050	194.188 ± 0.001
Finite size [a]	-0.036 ± 0.004 [b]	0.014 ± 0.003 [b]	0.005 ± 0.001 [b]	0.000 ± 0.000 [b]	-0.021 ± 0.002 [d]	-0.000 ± 0.000 [d]
Strong interaction					0.500 ± 0.050 [d]	0.008 ± 0.001 [d]
Vacuum polarization $\alpha(\alpha Z)$ [e]	1.609	1.555	0.917	0.486	1.425 [d]	0.794 [d]
Vacuum polarization $\alpha^2(\alpha Z)$	0.011	0.011	0.006	0.003	0.010	−0.006
Vacuum polarization $\alpha(\alpha Z)^3$	−0.011	−0.011	−0.005	−0.005	−0.006	−0.006
Vacuum polarization $\alpha(\alpha Z)^5$, $\alpha(\alpha Z)^7$	−0.001	−0.001	−0.000	−0.000	−0.000	−0.000
Lamb shift	0.000	0.000	0.000	0.000	0.000	0.000
Electron screening [f]	−0.013	−0.013	−0.010	−0.023	−0.005 [d]	−0.014 [d]
Nuclear polarization	0.005 ± 0.002	0.004 ± 0.001	0.001 ± 0.000	0.000 ± 0.000	0.004 ± 0.001	0.001 ± 0.000
Anomalous mag. moment of μ^-	0.006 ± 0.002	-0.004 ± 0.001	0.003 ± 0.001	-0.001 ± 0.000		
Pion polarization					0.001 ± 0.000	0.000 ± 0.000
Measured energy					278.069 ± 0.025	194.184 ± 0.016

[a] The following c, t-parameters were used: for Y $t=2.3$ and $c=4.86 \pm 0.10$ (Ref. [16]) and for Cd $t=2.3$ and $c=5.3804 \pm 0.0024$ (Ref. [17]).
[b] Included in the numerical solution of the Dirac equation.
[c] The calculations of the pionic transitions were performed assuming $m_\pi = 139.570$ MeV.
[d] Included in the numerical solution of the Klein-Gordon equation.
[e] Includes ladder graphs.
[f] Calculated with 2 K and 8 L electrons. Errors of the screening are not given (see Section 4).

energies making use of the proportionality between transition energy and pion mass for small variations. The transition energies were calculated with a computer code written by Krell and Tauscher [18–20] which solves the Klein Gordon equation numerically for a potential including contributions due to finite size, vacuum polarization in first order in αZ, strong interaction and electron screening. The parameters of the strong interaction pion-nucleus potential were taken from Tauscher [21]. An error of 10% was assumed for the strong interaction correction. The energies of the two pionic transitions were calculated with the assumed value of 139.570 MeV for the pion rest mass. The energies and all corrections are summarized in Table 2. The corrections for vacuum polarization, Lamb shift and nuclear polarization were performed as described in Section 3. The pion polarizability was calculated in the approximation of Ericson and Hüfner [13].

The electron screening as listed in Table 2 was calculated assuming that all K and L electrons are present during the muonic and pionic transitions. New measurements of the muonic cascade [22] in In indicate that about one K electron and four L electrons are present during the muonic $4f-3d$ and $5g-4f$ transitions. This would mean that the electron screening corrections have to be reduced by a factor of 2. Since one can assume that this reduction is similar for muons and pions, the π mass values were calculated with full screening for the muonic and pionic transitions. The π mass value without screening was 2.6 keV larger. Hence, assuming 50% screening, one has to add 1.3 keV to the π mass value calculated with full screening. The estimated screening error of 1 keV may be neglected compared to the final error of 10 keV. The π^- rest mass derived from the transitions in Y and Cd is:

139.575 ± 0.020 MeV from the $4f-3d$ transition in Y and

139.569 ± 0.012 MeV from the $5g-4f$ transition in Cd.

The averaged value of this experiment is

139.571 ± 0.010 MeV.

The error was determined from the uncorrelated experimental errors and the correlated errors of the evaluation procedure and the calibration method.

The value of this experiment together with previous results and a new averaged value of all measurements is given in Table 3. One has to keep in mind

Table 3. Compilation of various values for the π^- mass.

π^- mass (MeV)	Reference
139.37 ± 0.20	Growe and Philips (1954) [23]
139.566 ± 0.010	Shafer (1967) [1, 2]
139.569 ± 0.008	Backenstoss et al. (1971) [3, 4]
139.5657 ± 0.0024 [a]	Marushenko et al. (1976) [5]
139.5721 ± 0.0021	Mes et al. (1976) [6]
139.571 ± 0.010	this work
139.569 ± 0.002	averaged value

[a] An error of 1.8 keV [7] for the conversion of the pionic X-ray energy to the pion mass added to the purely experimental error of 1.7 keV given in Ref. [5].

that the errors of the different experiments are correlated to some extent by using the same fundamental constants and similar correction terms.

5. The μ-Neutrino Rest Mass

An upper limit of the μ-neutrino rest mass m_ν was calculated [24] with the averaged value of the π^- mass from all direct measurement (Table 3), $m_\pi = 139.569 \pm 0.002$ MeV, the tabulated value [25] of the μ mass, $m_\mu = 105.6595 \pm 0.0003$ MeV, and the averaged value [26–30] of the μ momentum from π^+ decay at rest, $p_\mu = 29.7882 \pm 0.0045$ MeV/c. A value of $m_\nu^2 = 0.15 \pm 0.36$ MeV^2 was obtained; the stated error is the standard deviation σ. In order to obtain an upper limit on m_ν we have to calculate

$$\Phi(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-t^2/2\sigma^2}\,dt \qquad (1)$$

for that x value which corresponds to the desired confidence level; note that Eq. (1) is not the usual error function as we exclude by physical reasons negative values of m_ν^2 which may be obtained as a result of experiments. This procedure yields at 90% confidence $m_\nu^2 < 0.61$ MeV^2 and $m_\nu < 0.78$ MeV. This is smaller than all other experimental results published.

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